MINIMIZATION OF MULTIPLICATIVE LABELING FOR SOME FAMILIES OF GRAPHS

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ABSTRACT

In this paper, we discuss minimization of multiplicative labeling for some families of Graphs. A function f is called a minimization of multiplicative labeling of a graph G with q edges, if f is a bijective function from the vertices of G to the set $\{1,2,3,.......p\}$ such that when each edge uv is assigned the label $f(uv) = f(u)*f(v) - \min\{f(u), f(v)\}$, then the resulting edge labels are distinct numbers. We investigated some families of graphs such as slingshot, stair, stethoscope, spectacles which admits minimization of multiplicative labeling.

KEYWORDS: Graph labeling, Multiplicative graph labeling, Graceful labeling.

1.1 INTRODUCTION

The field of mathematics plays a very important role in different fields of Science and Engineering. One of the most important areas in mathematics is graph theory. In graph theory, one of the main concepts is graph labeling. A graph can be labeled or unlabeled. Labeled graph are used to identification. Labeling can be used not only to identify vertices or edges, but also to signify some additional properties depending on the particular labeling. Graph labeling is an assignment of integer, to its vertices or edges subject to some certain conditions.

All graphs in this paper are finite and undirected. The symbols V (G) and E (G) denotes the vertex set and edge set of a graph G. A dynamic survey on graph labeling is regularly updated by Gallian [3] and it is published by Electronic journal of combinatories. Some basic concepts are taken from Frank Harary [4]. Shalini and Paul Dhayabaran [9] introduced the concept of minimization of multiplicative labeling. In this paper, we investigated some families of graphs such as slingshot, stair, stethoscope, spectacles which admits minimization of multiplicative labeling.

Definition 1.1

Let G = (V (G), E (G)) be a graph. A graph G is said to be minimization of multiplicative labeling if there exists a bijective function $f : \mathrm{V(G)} \to \{1,2,3,......\mathrm{p}\}$ such that, when each edge uv is assigned the label $f(uv) = f(u)*f(v) - \min\{f(u), f(v)\}$, then the resulting edge labels are distinct numbers.

Definition 1.2

A graph G is said to be minimization of multiplicative graph if it admits a minimization of multiplicative labeling.

2.1 Main Results

Theorem 2.1

The Slingshot $Slgt_n$ is a minimization of multiplicative graph.

Proof:

Let G be a graph of Slingshot $Slgt_n$

Let $\{v_1, v_2, v_3, \ldots, v_n, v_{n+1}\}$ be the vertices of $Slgt_n$ and $\{e_1, e_2, e_3, \ldots, e_n, e_{n+1}\}$ be the edges of $Slgt_n$ which are denoted as in the Figure 2.1

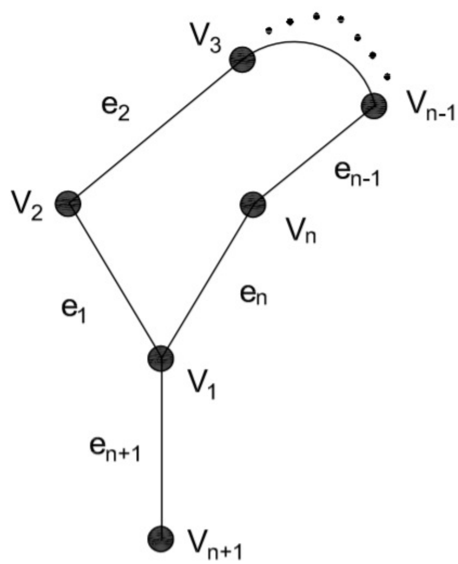

Fig.2.1: Slingshot $Slgt_n$ with ordinary labeling

The Slingshot $Slgt_n$ consists of n + 1 vertices and n + 1 edges.

The vertices of $Slgt_n$ are labelled as given below

Define $f : V(G) \rightarrow \{1,2,3,\ldots,n+1\}$ by

$f(v_i) = i + 1 \; ; \; 1 \leq i \leq n$

$f(v_{n+1}) = 1$

Then the edge labels are:

$f(e_i) = (i + 1)^2 \; ; \; 1 \leq i \leq n - 1$

$f(v_1 v_n) = 2n$

$f(e_{n+1}) = 1$

The edges of the slingshot graph receive distinct numbers.

Hence, the slingshot $Slgt_n$ is a minimization of multiplicative graphs.

Example: 2.1

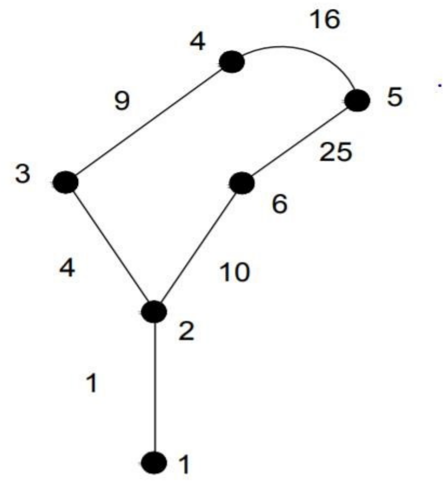

Fig.2.2: Slingshot $Slgt_5$

Theorem 2.2

The graph Stair Str_n is a minimization of multiplicative graphs.

Proof:

Let G be a graph of stair Str_n .Let $\{v_1, v_1, v_3,\ldots, v_n ,v_{n+1,}\ v_1^{'}, v_2^{'}, v_3^{'},\ldots, v_n^{'}\}$ be the vertices of Str_n and $\{e_1, e_2, e_3, \ldots , e_n, e_1^{'} , e_2^{'} , e_3^{'} ,\ldots, e_n^{'}\}$ be the edges of Str_n which are denoted as in the Figure.2.3

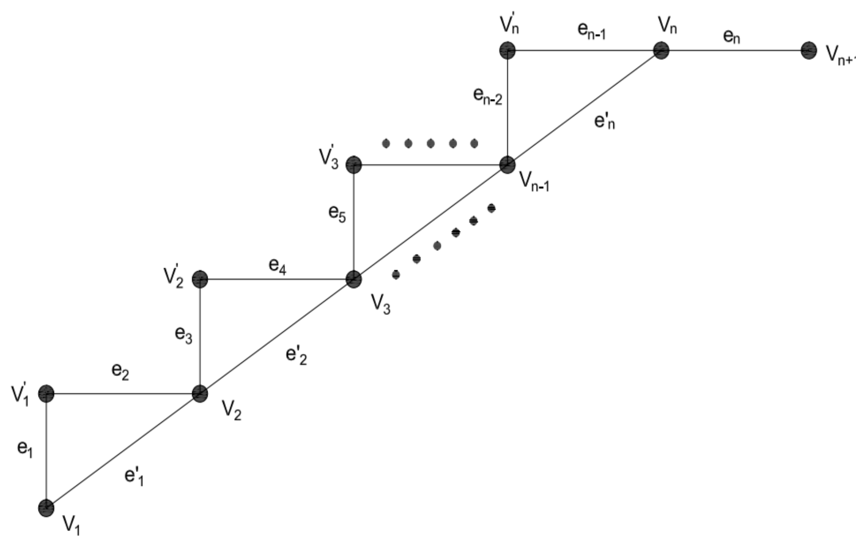

Fig.2.3 The Stairs Str_n with ordinary labeling

The graph Stairs Str_n consists of 2n vertices and 3n – 2 edges.

The vertices of Str_n are labelled as given below.
Define $f : V(G) \rightarrow \{1,2,3,\ldots,2n\}$ by

$f(v_i) = 2i - 1 \; ; \; 1 \leq i \leq n$

$f(v_i') = 2i \; ; \; 1 \leq i \leq n - 1$

$f(v_{n+1}) = 2n$

Then the edge labels are:

$f(e_i) = i^2 \; ; \; 1 \leq i \leq n - 1$

$f(e_i') = 4i^2 - 2i \; ; \; 1 \leq i \leq n-1$

The edges of Str_n graph receive distinct numbers.

Hence, Str_n is a minimization of multiplicative graphs.

Example: 2.2

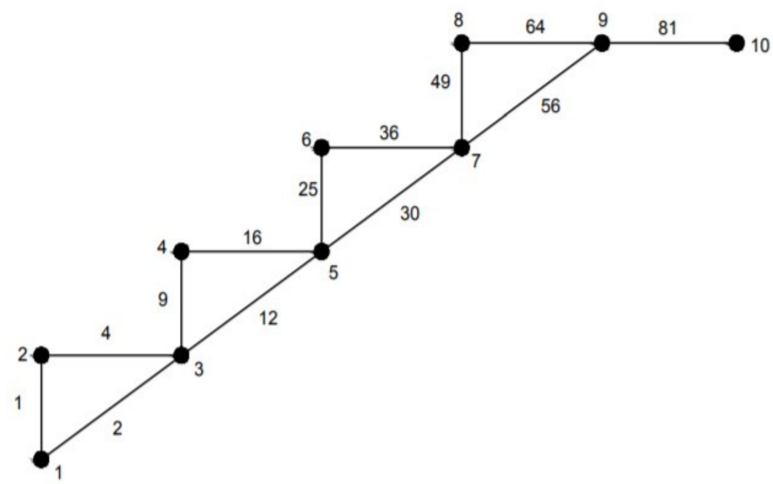

Fig.2.4 stair Str_5

Theorem: 2.3

The graph Stethoscope S_3 is a minimization of multiplicative graphs.

Proof:

Let G be a graph of Stethoscope S_3.Let $\{v_1, v_1, v_3,\ldots, v_n ,v_{n+1}\}$ be the vertices of S_3 and $\{e_1, e_2, e_3, \ldots, e_n ,e_{n+1}\}$ be the edges of S_3 which are denoted as in the Figure.2.5

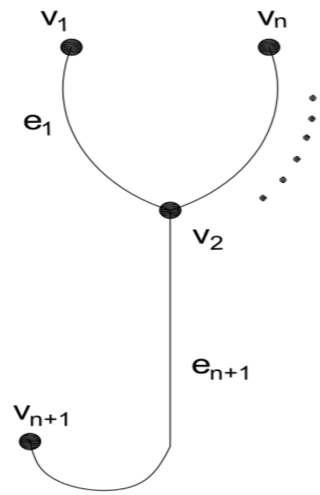

Fig.2.5 The Stethoscope S_3 with ordinary labeling

The graph Stethoscope S_3 consists of n vertices and n edges.

The vertices of S_3 are labelled as given below.
Define $f : V(G) \to \{1,2,3,\ldots,n\}$ by

$f(v_i) = i \; ; \; 1 \le i \le n$

$f(v_{n+1}) = n+1$

Then the edge labels are:

$f(e_i) = i^2 \; ; \; 1 \le i \le n - 1$

$f(a) = uv\text{-}\min\{u,v\}$

The edges of S_3 graph receive distinct numbers.

Hence, S_3 is a minimization of multiplicative graphs.

Example: 2.3

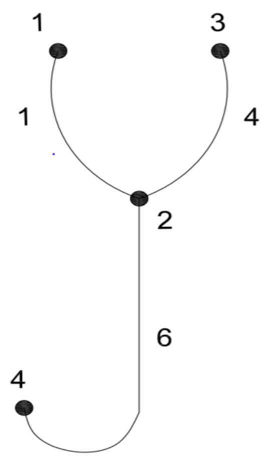

Fig.2.6 Stethoscope S_3

Theorem: 2.4

The graph Specs $K_2 \odot C_n$ is a minimization of multiplicative graphs.

Proof:

Let G be a graph of Specs $K_2 \odot C_n$

Let $\{v_1, v_2, v_3, \ldots, v_n, , v_1', v_2', v_3', \ldots, v_n'\}$ be the vertices of Specs $K_2 \odot C_n$ and $\{ e_1, e_2, e_3, \ldots, e_n, e_{n+1}, e_1', e_2', e_3', \ldots, e_n'\}$ be the edges of Specs $K_2 \odot C_n$ which are denoted as in the Figure. 2.7

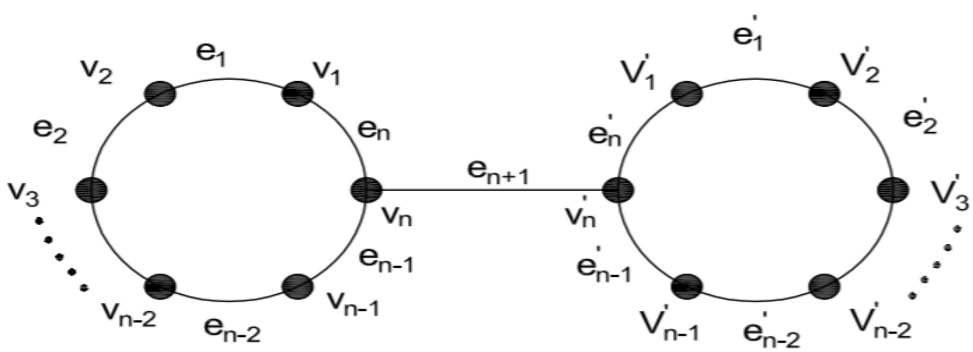

Fig.2.7.The Specs $K_2 \odot C_n$ with ordinary labeling

The Specs $K_2 \odot C_n$ consists of 2n vertices and 2n+1 edges.

The vertices of Specs $K_2 \odot C_n$ are labelled as given below.

Define $f : V(G) \to \{1, 2, 3, \ldots, 2n\}$ by

$f(v_i) = i \; ; \; 1 \leq i \leq n$

$f(v_i') = n + i \; ; \; 1 \leq i \leq n$

Then the induced edge labels are:

$f(e_i) = i^2 \; ; \; 1 \leq i \leq n - 1$

$f(e_n) = n - 1$

$f(e_i') = n + i \; ; \; 1 \leq i \leq n - 1$

$f(e_n') = 2n^2 + n - 1$

$f(e_{n+1}) = n$

The edges of $K_2 \odot C_n$ receive distinct even numbers.

Hence, Specs $K_2 \odot C_n$ is a minimization of multiplicative graphs.

Example: 2.4

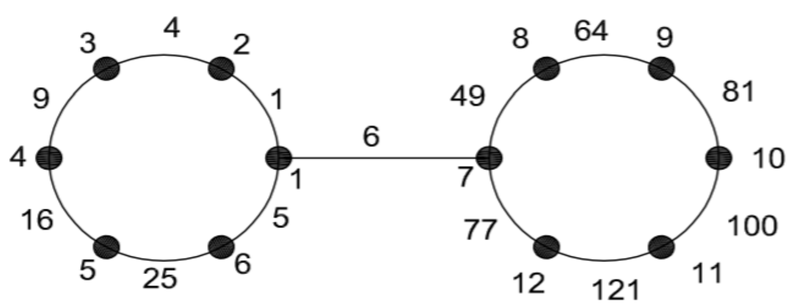

Fig.2.8 $K_2 \odot C_6$

Conclusion

In this paper, We discussed minimization of multiplicative labeling for some families of graphs. We investigated such as slingshot, stair, stethoscope, spectacles which admits minimization of multiplicative labeling.

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