# Middle Distance Dominating Set

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**Abstract**

A distance dominating set D of V in $G = (V, E)$ is said to be **M**iddle **D**istance **D**ominating set (MDD) if $\forall \ u_i \in D \ such \ that \ \ d(u_i, v_j) \leq k \quad v_j \in V - D$ where k is number of vertices in D and the cardinality of G is $\gamma_M(G) = k$, in this article show that the MDD is more effective than some other dominating set and how to construct the MDD set for the graph G using Radius algorithm and diameter algorithm. Finely we find unique dominating set for a graph G is as in middle of graph G.

**Keywords:** Distance dominating set, Middle distance dominating set, cut vertex.

## 1. Introduction

The history of graph theory has been introduced since 1960, domination has been introduced since 1970 [8], the middle of the year so many domination were introduced by many author and recently some dominations like Total Domination, Perfect Domination, Fair Domination, Secure Domination, Power Domination, Restrained Domination, Locating Domination, Super Domination, Convex Domination and etc. all domination has some unique characterization and their application are very use full to the problems involving locating optimally a hospital, police station, fire station, or any emergency service facility and construct a route for a school bus that leaves the school and resource allocation and placement in parallel for given network.

In this paper focused domination should be in center for any distribution center in business, farming products are sale in nearby place, planning to construct new hospital, school, bus stand, airport, railway station, SIPCOT in Tamilnadu (India), theatre and etc. in this connection we have to need some basic dimension in graph theory, so, we have to use the radius and diameter of the graph to finding domination center through radius algorithm and diameter algorithm and implemented the distance domination concept in those algorithm. More ever we have finding middle dominating path for route related problems, it will be very useful to distribute the item to various nearby places by minimum cost and distance, in a network, there are some many path available but there are not comfortable than middle distance dominating path because our path act as either good ratio partitions or path vertices are maximum degrees [2,3,5]. In any graph $G = (V, E)$, a vertex v is said to be cut vertex if remove the vertex v from G such that G is disconnected, the minimum distance between two nonadjacent vertices is called radius of the graph G, the maximum distance between two vertices is called diameter of a graph G, the neighbors of the vertex u in V denoted by N(u) such that N(u) is collection of all adjacent vertices of u in G, the corona graph of a graph G is defined by each vertex of graph G has a same graph G which is place at nearby and each vertices such that $\forall\ v_i \in G \Rightarrow d(v_i, v_j) = 1$ *for all nearby graph* $v_j \in G$, it is clear that if G has n vertices then corona of G has n(n+1) vertices, a path P in G is sequence of vertices and edges such that P is simple unique connected graph.

## 2. Definitions

**Definition 2.1:** A distance dominating set D of V in $G = (V, E)$ is said to be ***Middle Distance Dominating set*** $(MDD)$ if $\forall\ u_i \in D$ *such that* $d(u_i, v_j) \le k$ *each* $v_j \in V$ where k is number of vertices in D or the cardinality of G is $\gamma_M(G) = k$,

**Definition 2.2:** A $path\ P = (u, v)$ in a graph G has n vertices is said to be ***Middle Distance Dominating Path*** if each vertices of P dominated all other vertices at the distance less than or equal to $d = \frac{n}{2^k}$, where k is number of vertices in path P

**Definition 2.3:** A radius distance dominating path $P = (u, v)$ in a graph G is said to be ***Divider Distance Dominating Path*** if P is cut path.

**Definition 2.4:** A graph G has ***Radius Rule*** if a vertex set of V in G has k vertices are dominated all other vertices at most k distance. That is, if k=1, such a vertex dominated all other vertices at most distance one. If k=2, such a pair of vertices are dominated all other vertices at most distance two. If k=3, such a triple of vertices are dominated all other vertices at most distance three. It is easy to obtain such a vertex set by removing the boundary vertices at each stage.

**Lemma 2.1: Let G be a simple connected graph on n vertices, if k be the number of vertices in path dominating set P, then P is dominated all other vertices at most distance of k.**

**Radius Algorithm:**

Step 1: Choose a maximum degree (say x) of vertices from graph G

Step 2: Say$\{v_1, v_2, v_3, \ldots v_m\}$has degree x and find degrees to all adjacent vertices of each $v_i$

Step 3: More number of adjacent vertices of each $v_i$ has same degree x or degree < x

Step 4: Create a path with $P_1=(v_i,v_j)$ where $v_j$ are adjacent to $v_i$

Step 5: The path $P_1$ has two vertices, these two vertices are dominated all other vertices at a distance two.

Step 6: if yes,$\{ v_i,v_j \}$ are radius distance dominating set of the graph G, then stop, otherwise go to step 1( try and choose next maximum degree vertex)

**Diameter Algorithm**

Step 1: Choose path P as a radius distance dominating path

Step 2: if P is divider distance dominating path. Stop, otherwise go to step 3

Step 3: add a vertex with adjacent vertices of P, the new Path $P_1$ is divider distance dominating path. Stop, otherwise continue same process, and up to obtain a cut path

Step 4: check the cut path as one of the comfortable divider distance dominating path

Step 5: cut path is comfortable,

if either

1. Remove cut path from G, the collection of the partitions are nearly equal (or)
2. Cut vertices of path are dominating more number of vertices than other path

## 3. Middle Distance Domination on Tree

A tree T is connected without cycle graph [1,4,6,7] and any two vertices are connected by a unique path, in that number of edges is equal to number of vertices minus one, A tree has at least two pendant vertices if v > 1. Let T be a tree on n vertices,

if $v \in T$ degree of v is n-1 such that {v} is middle distance dominating set since radius rule satisfied, also $d(v,v_i)=1, \ \forall v_i \in T$ by theorem 5.1[9], more ever v is cut vertex and it is trivial divider.

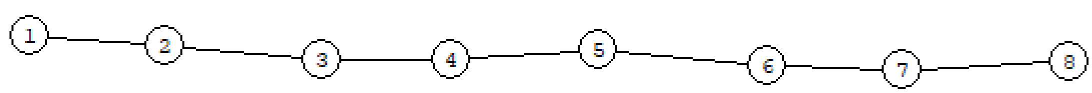
if $v \in T$ degree of v is n-2 such that the degrees of $\{v_1,v_2\}$ are 4,2 respectively $v_1$ and $v_2$ are adjacent to each other by theorem 5.2[9] and proposition 3.1[9] says there is 4 pendant vertices, therefore there exist a path $P(v_1,v_2)$ and this path P is middle distance dominating path and divider also.

if $v \in T$ degree of v is n-3 such that the degrees of $\{v_1,v_2,v_3\}$ are 3,2,2 respectively. by theorem 5.3[9] and proposition 3.1[9] says there is 3 pendant vertices, therefore there exist a path $P(v_1,v_2,v_3)$ and this path P is middle distance dominating path and divider also.

We continue in way, if $v \in T$ degree of v is 2 as maximum such that the degrees of $\{v_1,v_2,\ldots,v_{n-2}\}$ are 2,2,….,2 respectively. Since a tree has at least two pendant vertices if v > 1it says that there is 2 pendant vertices, therefore there exist a path $P(v_1,v_2,\ldots\ldots,v_{n-2})$ and this path P is middle distance dominating path and divider also.

**Question3.1:** A Tree has two vertices, What about middle distance dominating set?

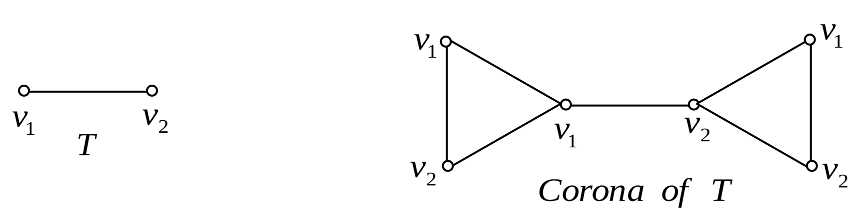
**Example 3.1:** Consider a tree on 8 vertices.




(1) The middle distance dominating set {4}, {5} and $k = 1$ ,Since $d = \frac{n}{2^k} = \frac{8}{2} = 4$

(2) The middle distance dominating path and divider {2,3,4,5,6,7} and $k = 6$

**Example 3.1:** Consider a corona of tree T on 2 vertices

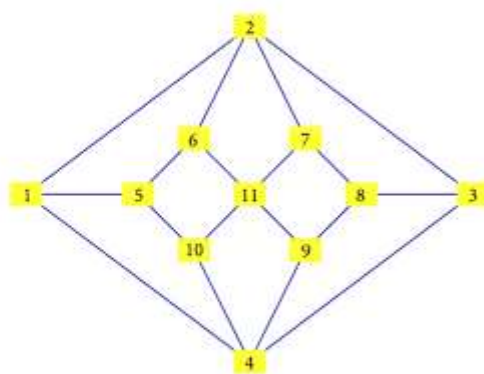


The vertex set $\{v_1,v_2\}$ of T is a path $P(v_1,v_2)$ in corona of T which is middle distance dominating set, path and divider.

## 4. Middle Distance Domination on Graphs

Theorem 3.1 [6] says, the connectivity $\kappa(G)$ of G is always less than or equal to minimum degree of G for any graph G. The divider distance dominating path has not minimum cardinality due to the condition $\kappa(G) = k \geq \delta$ where $\delta$ is minimum degree of G. clearly some of familiar path like Hamilton path is not a divider distance dominating path. Complete graph has not divider distance dominating path, we may take it as an answer of the question 3.1. The Herschel graph has maximum degree is equal to its length of divider distance dominating path, since Herschel graph has 11 vertices, the path takes to cover the vertices from its boundary and middle of the graph.

The cut vertex set are {2,4,11} and {1,3,5,8,11}, its cut path are $P_1$=(2,6,11,10,4) or (2,7,11,9,4) and $P_2$=(1,5,10,11,9,8,3) or (1,5,6,11,7,8,3), each cut path having boundary vertices of graph but they are not adjacent vertices, length of $P_1$ and $P_2$ are 4 and 6 respectively. Its maximum degree 4, so we can chose the path $P_1$ for divider distance dominating path. Due to length of the path not exceed it maximum degree and each vertices are dominated all other vertices at most 3 distance.

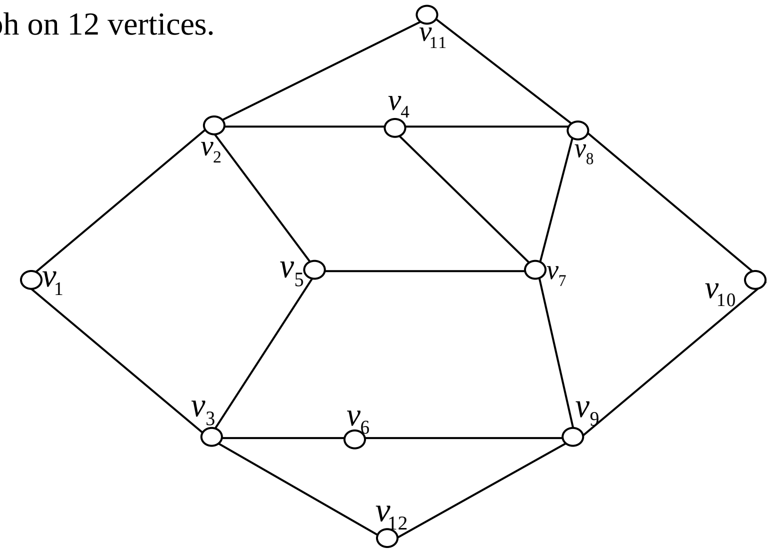



Herschel Graph

In this paper, mainly focus that, more number of vertices are dominating by minimum number of vertices. So, in Herschel graph $P_1$ is dominating 6 vertices and $P_2$ is dominating 4 vertices. Therefore we chose the path $P_1$ as a divider distance dominating path.

**Example 4.1:**

Consider the Graph on 12 vertices.




From the example 4.1

(1) Let P={$v_3$,$v_6$}, if n=12 and k=2

Here the distance of the domination less than or equal to 12/4=3

But $v_3$,$v_6$ dominated $v_{11}$ at the distance 3,4 respectively

So, P is not middle distance dominating path.

(2) Let P={$v_2$,$v_4$,$v_8$}, if n=12 and k=3

Here the distance of the domination less than or equal to 12/8=1.5

But $v_2$,$v_4$,$v_8$ are dominated $v_6$ at a distance 3,3,3 respectively

So, P is not middle distance dominating path.

(3) Let P={$v_5$,$v_7$}, if n=12 and k=2

Here the distance of the domination less than or equal to 12/4=3

So, P is middle distance dominating path.

(4) Let P={$v_4$,$v_7$}, if n=12 and k=2

Here the distance of the domination less than or equal to 12/4=3

So, P is middle distance dominating path.

**Using diameter algorithm, to finding divider distance dominating path for the example 4.1.**

Let G be simple connected graph on 12 vertices.

We can choose the vertices of maximum degree $\{v_{2,}v_{3,}v_{7,}v_{8,}v_9\}$ has degree 4

$N(v_2)$ are $\{v_1,v_4,v_5,v_{11}\}$ its degrees are {2,3,3,2} respectively

$N(v_3)$ are $\{v_1,v_5,v_6,v_{12}\}$ its degrees are {2,3,2,2} respectively

*$N(v_7)$ are $\{v_4,v_5,v_8,v_9\}$ its degrees are {3,3,4,4} respectively*

$N(v_8)$ are $\{v_6,v_7,v_{10},v_{11}\}$ its degrees are {2,4,2,2} respectively

$N(v_9)$ are $\{v_4,v_7,v_{10},v_{12}\}$ its degrees are {3,4,2,2} respectively

The vertex $v_7$ has $v_2$ vertices has same degree as 4

So, choose a path $\{(v_7,v_4), (v_7,v_5), (v_7,v_8), (v_7,v_9)\}$

The each vertex of path $(v_7,v_4)$, $(v_{7,5})$, $(v_7,v_8)$ and $(v_7,v_9)$ are not dominated all other vertices at a distance two. Since $N(v_7)$ are $\{v_4,v_5,v_8,v_9\}$ so, any two vertices add with $v_7$ that path are dominated all other vertices at the distance 3. We chose if the path $(v_7,v_4,v_5)$ is radius distance dominating path, if the path $(v_7,v_8,v_9$ ) is radius distance dominating path and also divider, $v_8$ and $v_9$ are boundary and $v_7$ is middle. Since length of path $(v_7,v_8,v_9)$ is 3 are dominating 9 number of vertices. if there exist any other path act as divider distance dominating path it will be same effect as Path $(v_7,v_8,v_9)$ but not comfortable to dominating vertices.

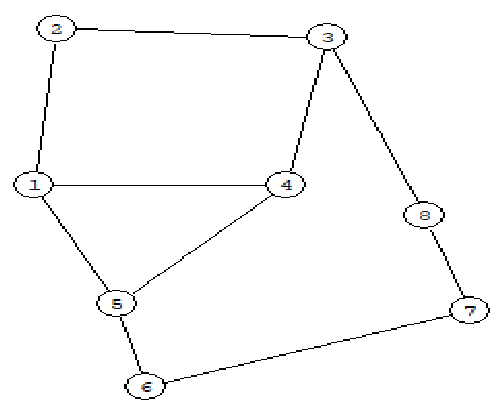
**Question 4.1:** why the path $(v_7,v_8,v_9)$ is comfortable than other path?

**Example 4.2:**

The number of vertices $n = 8$, middle distance dominating path $\{each\ vertex\}$ and $k = 1$,

Since $d = \frac{n}{2^k} = \frac{8}{2} = 4$

The number of vertices $n = 8$, the radius distance dominating path {3,4,5} and $k = 3$, and also divider

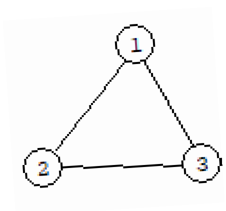



**A Graph on 8 vertices**

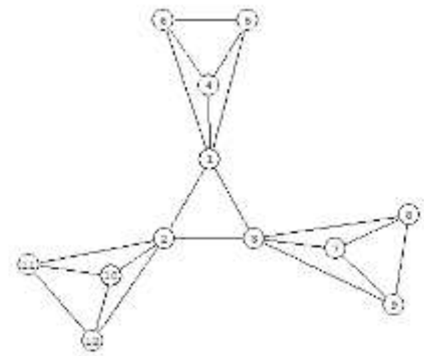
**Question 4.3:** why the path (3,4,5) is comfortable than the path (1,4,5)?

**Example 4.3:**

Any corona graph has middle, radius, and divider distance dominating path




Graph G



Corona Graph of G

**5. Middle Distance Domination on Some Dominating Sets**

The characteristic of middle distance dominating set is actually sequence of adjacent vertices, so we focus on path (sequence of adjacent vertices) in some dominating sets, we investigate the existing of path on every dominating sets and its some properties[10,11,12,13,14,15]

**Proposition 4.1:** Every perfect dominating set has at least one path.

**Proposition 4.2:** Every cut path in perfect dominating set is middle distance dominating path.

**Proposition 4.3:** Every middle distance dominating path has at least one vertex from in perfect dominating set.

**Proposition 4.4:** Every fair dominating set has at least one path.

**Proposition 4.5:** Every fair dominating set has need not path.

**Proposition 4.6:** Every fair dominating set has need not middle distance dominating path.

**Proposition 4.7:** Every locating dominating set has either path or not path.

**Proposition 4.8:** Every locating dominating set has at least one path on tree.

**Proposition 4.9:** Every restrained dominating set has need not path.

**Proposition 4.10:** Every secure dominating set has need not path.

**Proposition 4.11:** Every super dominating set has need not path.

## 6. Conclusion:

In this paper, we focus a unique and best path in any graph G, this path let help the networking problem for various reasons. There are many domination in graph theory history, but this middle distance domination is comparably best than other domination. The discussion of the path is shows that how to differ the path on trees, general graphs and some dominating sets.

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